that the vortex formation frequency and thus the drag coefficient, by reason of the present article, can be changed by the action of an external acoustic signal.

Additional remarks should await a thorough experimental investigation.

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Dynamic Response of a Nonlinear Membrane in Supersonic Flow

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Nomenclature

a = membrane length
E = modulus of elasticity
h = membrane thickness
M = Mach number
n = number of grid points
q = dynamic pressure, $\rho U^2/2$
t = time
T = membrane tension
T_0 = membrane pretension
U = freestream velocity
V_0 = initial velocity of membrane
w = membrane deflection
w_0 = initial displacement of membrane
x = coordinate parallel to airflow
β = $(M^2 - 1)^{1/2}$
γ = membrane mass per unit area
λ = dynamic pressure parameter, $2qa/\beta T_0$
ρ = mass density of air
ω = circular frequency

Introduction

SEVERAL investigators in the field of panel flutter have given cursory attention to a degenerate case of a panel (the membrane) and have concluded that, based on linear theory, a membrane is stable when subjected to supersonic airflow over one surface. On the other hand, experimentalists have noted that panels that closely approximate membranes appear to flutter physically.

In view of this apparent experimental contradiction of the analytical prediction, there is some question concerning the effect of nonlinearities due to large deflections on the dynamic behavior of a membrane in a gas flow. The purpose of this note is to consider the nonlinear problem as an initial value problem in order to determine the effect of the nonlinearities.

Analysis

Statement of problem

An infinitely wide membrane of length a in the flow direction (x) is considered (see Fig. 1). When linearized static aerodynamic strip theory is utilized, as in Ref. 1, the nonlinear differential equation, together with the appropriate boundary and initial conditions, are

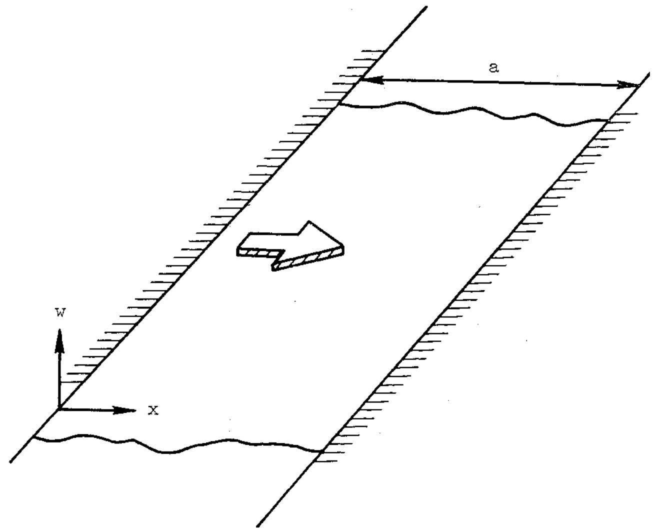

Fig. 1 Membrane geometry and coordinate system.

$$w_{xx} - \frac{2q}{\beta T} w_x - \frac{\gamma}{T} w_{tt} = 0 \qquad \left.\begin{aligned} w(0,t) &= w(a,t) = 0 \\ w(x,0) &= w_0(x) \\ w_t(x,0) &= V_0(x) \end{aligned}\right\} \tag{1}$$

where

$$T = T_0 + \frac{Eh}{2a} \int_0^a (w_x)^2\, dx \tag{2}$$

Equation (2) expresses the fact that the tension in the membrane is a function of the deflected shape and is independent of x since inplane inertia is neglected.

Solution

Linear case, exact solution. For the linear case an exact solution is

$$w = \sum_{m=1}^{\infty} A_m e^{\lambda x/2a} \sin\frac{m\pi x}{a} \cos\omega_m t \tag{3}$$

where

$$\omega_m{}^2 = \frac{T_0}{a^2\gamma}\left[\left(\frac{\lambda}{2}\right)^2 + (m\pi)^2\right] \tag{4}$$

$$A_m = 2\int_0^a e^{-(\lambda x/2a)}\, w_0 \sin\frac{m\pi x}{a}\, dx \tag{5}$$

and $V_0 = 0$.

The series (3) is seen to converge providing the series for the case $\lambda = 0$ converges. There are no values of λ which will cause an infinite displacement from a finite input; thus, for the linear case, there can be no critical value of the dynamic pressure parameter from the classical point of view, since there are no values of λ which will make ω_m [Eq. (4)] complex.

Finite-difference solution. In order to investigate the effects of the nonlinear stiffness of the membrane, a numerical procedure was used. Both the space and time derivatives in Eqs. (1) and (2) were approximated at a discrete number of points and times, respectively. The resulting set of equations was then integrated numerically to provide a time history of the response.

The total membrane energy, i.e., the sum of the kinetic and potential energies of the membrane, was used as a criterion of stability. The system was assumed to be stable if the response of the membrane energy with time were bounded, and unstable if the converse were true. Prior investigation by

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Received September 16, 1964.
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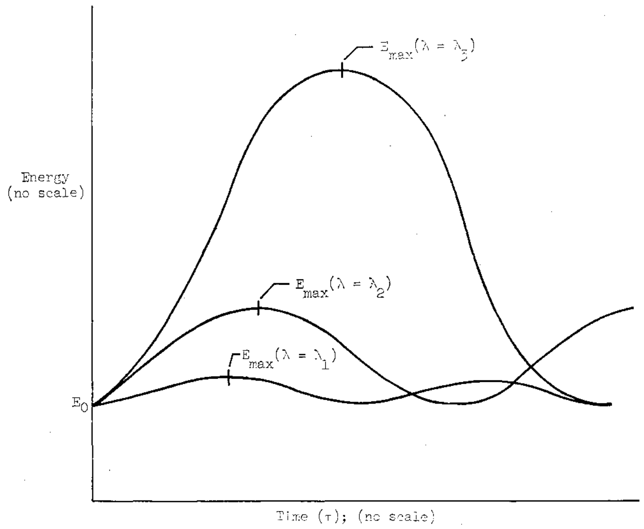

Fig. 2 Membrane system energy vs time. ($\lambda_3 > \lambda_2 > \lambda_1$).

the authors of the energy response of a plate subjected to supersonic gas flow had shown that the total plate energy response can be used to indicate stability.

A typical plot of the total membrane energy vs time, for the nonlinear case considering no damping, is shown in Fig. 2. The initial energy E_0, which is associated with the statically deflected membrane of shape w_0, is seen to be magnified by the action of the airflow to a maximum value E_{max}. Increasing the value of dynamic pressure parameter λ as represented by λ_1, λ_2, and λ_3 was found to give a similar energy response, for given initial conditions, except that the period of oscillation and the maximum value of the energy were changed. The change of the maximum energy with the dynamic pressure parameter λ is shown in Fig. 3. The maximum membrane energy was found to vary approximately exponentially with λ, but no instability was found.

The finite-difference equations associated with the linear case ($T = T_0$) were also solved by the same numerical integration procedure used in the nonlinear case. For the linear case a critical value of the dynamic pressure parameter was found to be associated with each number of grid points n. At this critical value of λ the system energy associated with the linear solution increased without bound; for the same value of λ, the corresponding solution for the nonlinear problem showed no departure from the energy response characteristics presented in Fig. 2, indicating that the nonlinearities are stabilizing. The critical value of λ_n associated with a particular number of grid points increases with n

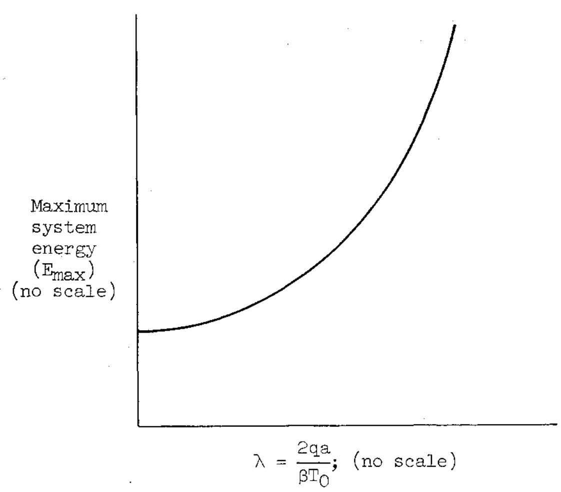

Fig. 3 Variation of maximum system energy with dynamic pressure parameter, λ.

and was found to approach infinity as n approaches infinity. It is of interest that a Galerkin solution to the linearized membrane problem also gives a critical value of λ for a finite number of modes. As was pointed out in Ref. 2, the critical value of λ approaches infinity as the number of modes approaches infinity.

Concluding Remarks

In this study, the response of a membrane in supersonic flow has been considered as an initial value problem. The results show that the effect of the nonlinearity (increasing tension with deflection) is stabilizing from the standpoint of classical stability for all finite values of the dynamic pressure for the case of a finite number of grid points. Both the period and magnitude of the response were reduced compared to the linear solution for a given value of the dynamic pressure parameter λ. However, from observation a membrane can appear to be fluttering, since the magnitude of an initial disturbance is magnified by an exponential function of λ and x. The experimental determination of any flutter behavior of membrane-like panels may thus be difficult, since flow disturbances can cause a response that resembles the classical flutter behavior.

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Flow Field for Sonic Jet Exhausting Counter to a Hypersonic Mainstream

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Nomenclature

A	= area
C_D	= drag coefficient
D_j	= distance of jet shock from nozzle exit
M	= Mach number
p	= pressure
q	= dynamic pressure
R	= radius
T	= thrust
U	= velocity
ρ	= density
γ	= ratio of specific heats

Subscripts and Superscripts

1	= jet throat
2	= downstream jet station (see Fig. 1)
∞	= freestream conditions
i	= interface or contact surface between two flows
j	= jet conditions
t	= stagnation conditions
$(\ )'$	= conditions after a normal shock

INFORMATION reported in Ref. 1 has provided an explanation for two different types of mainstream shock displacements for jets exhausting counter to a mainstream flow.

Received September 14, 1964.

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